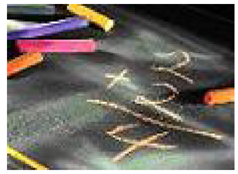

DETAILED SOLUTIONS AND CONCEPTS - RIGHT TRIANGLE TRIGONOMETRY
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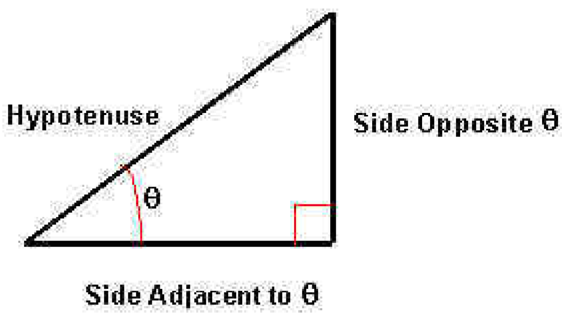
Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

The six trigonometric ratios of an **ACUTE** angle θ are defined as follows in a right triangle



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

The abbreviations "opp", "adj", and "hyp" represent the lengths of the three sides of the right triangle above.

opp = the length of the side opposite the angle θ
adj = the length of the side adjacent to the angle θ
hyp = the length of the hypotenuse

NOTE: SOHCAHTOA (sine equals opposite over hypotenuse; cosine equals adjacent over hypotenuse; and tangent equals opposite over adjacent) is a nice memory aid that might better allow you to keep the ratios in mind.

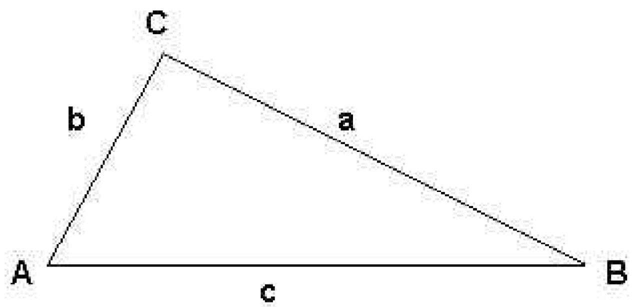

Doesn't SOHCAHTOA sound like a tropical island where we would rather be than in this course? hehehe

You can use trigonometric functions to model and solve real-life problems. The following is only an introduction to all the different applications for trigonometry.

NOTE 1: In a triangle the names of the vertices are often also used for angle names! For example, in the triangle ABC below, we can refer to angle "A", angle "B", and angle "C".

Another way of referring to angles uses all three vertices of the triangle. For example, in the triangle below, we could also name angle "A" as angle "BAC" or angle "CAB". Notice that the angle we are referring to must be in between the other two vertices!

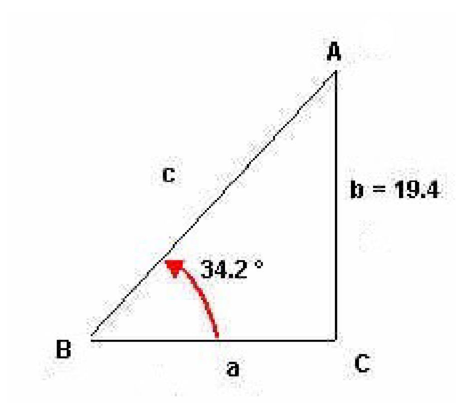
NOTE 2: For Examples 1 and 2, assume that angle "A" is opposite side "a", angle "B" is opposite side "b", and angle "C" is opposite side "c".



NOTE 3: During the process of solving for an unknown side or angle always use given values first. If you have to use a calculated value let's agree to round it to 2 decimal places.

Problem 1:

Solve the right triangle pictured below. That is, find all unknown sides and angles. Round the final solutions to one decimal place! Solve for angle ***A*** first, then for side ***a***, and finally for side ***c***.



Solve for Angle *A*:

Since the sum of the interior angles of a triangle always equals 180^o, we find that

$$A = 180^o - 90^o - 34.2^o = 55.8^o$$

Solve for Side *a*:

Let's use the fact that $\tan A = \frac{opp}{adj} = \frac{a}{b}$

In our case, $\tan 55.8^\circ = \dfrac{a}{19.4}$

and $a = 19.4 \tan 55.8^\circ \approx 28.5$

Solve for Side *c*:

Let's use the fact that $\sin B = \dfrac{opp}{hyp}$

In our case,

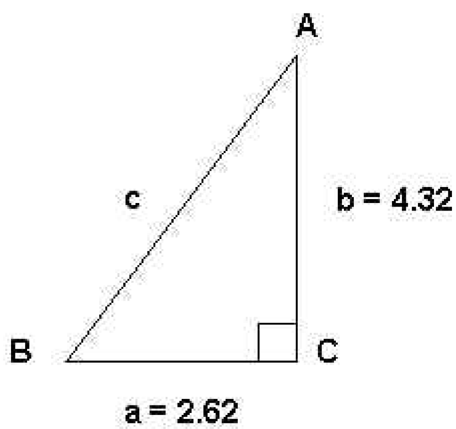
$$\sin 34.2^\circ = \frac{19.4}{c}$$
$$c \sin 34.2^\circ = 19.4$$

and $c = \dfrac{19.4}{\sin 34.2^\circ} \approx 34.5$

We find that $A = 55.8^o$, $a \approx 28.5$, and $c \approx 34.5$

Problem 2:

Solve the right triangle pictured below. Round the final solutions to two decimal places. Solve for angle ***A*** first, then for Angle ***B***, and finally for side ***c***.



Solve for Angle *A*:

Let's use the fact that $\tan A = \dfrac{opp}{adj} = \dfrac{a}{b}$

$$\tan A = \frac{2.62}{4.32}$$
$$A = \tan^{-1}\left(\frac{2.62}{4.32}\right)$$
$$A \approx 31.24^{\circ}$$

Solve for Angle *B*:

Since the sum of the interior angles of a triangle always equals ***180º***, we find that

$$B \approx 180^{\circ} - 90^{\circ} - 31.24^{\circ} = 58.76^{\circ}$$

Solve for Side *c*:

Since we are discussing a right triangle, we can use the *Pythagorean Theorem*, that is,

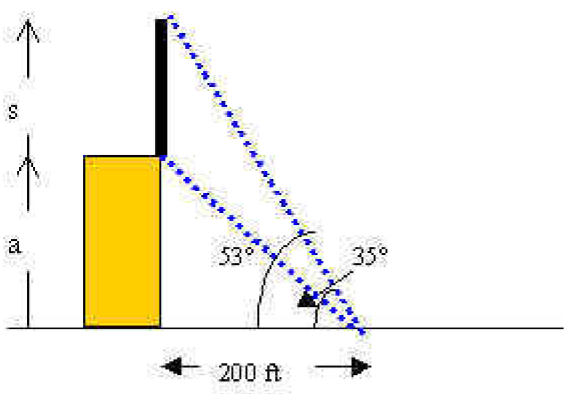
$$c^2 = (2.62)^2 + (4.32)^2$$
$$c = \pm\sqrt{(2.62)^2 + (4.32)^2}$$
$$c \approx \pm 5.05$$

Given the initial information, the triangle has the following solutions:

$A \approx 31.24^{\circ}$, $B \approx 58.76^{\circ}$, and $c \approx 5.05$ ***ft*** (remember that a distance is never negative!)

Problem 3:

At a point 200 feet from the base of a building, the angle of elevation to the bottom of an antenna is ***35º***, and the angle of elevation to the top is ***53º***, as shown in the picture. Find the height ***s*** of the antenna rounded to one decimal place.



Note from the picture that this problem involves two right triangles.

Use $\tan 35^{\circ} = \frac{a}{200}$ to conclude that the height of the building is $a = 200 \tan 35^{\circ}$.

Then use $\tan 53^\circ = \frac{a + s}{200}$ or $\tan 53^\circ = \frac{200\tan 35^\circ + s}{200}$

to conclude that the height of the antenna is

$$200\tan 53^\circ = 200\tan 35^\circ + s$$
$$s = 200\tan 53^\circ - 200\tan 35^\circ$$

Thus, the height of the antenna is approximately 125.4 feet.