DETAILED SOLUTIONS AND CONCEPTS - LOGARITHMIC FUNCTIONS

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PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Definition of Logarithmic Function

The logarithmic function is the inverse of the exponential function $y = b^x$, that is $x = b^y$.

Solving the inverse of the exponential function for ***y***, the logarithmic function is written as

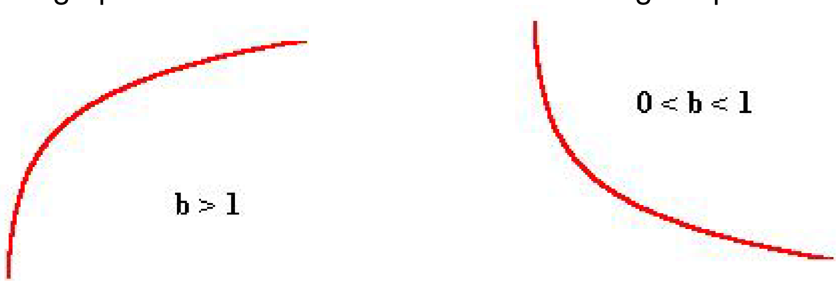
$y = \log_b x$ (pronounced ***log base b of x***), where ***b*** is a positive number other than ***1***

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

Characteristics of Graphs of Logarithmic Functions

Most logarithmic functions $f(x) = \log_b x$ and their transformations are best graphed with a graphing utility because the y-values get extremely large/small very quickly and are difficult to show in a hand-drawn *Cartesian Coordinate System*.

The graph of $f(x) = \log_b x$ has the following shapes:



- The graph consists of a SMOOTH curve with a rounded turn.
- Logarithmic functions and their transformations have *vertical asymptotes*.
- The equation of the *vertical asymptote* of the graph of $f(x) = \log_b x$ is ***x = 0***, which is the x-axis.

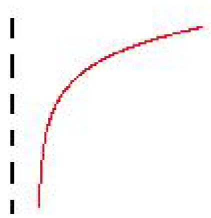
- ONLY horizontal shifts of the graph of $f(x) = \log_b x$ change the equation of the *vertical asymptote*.
- The graph is never parallel to the x-axis, but moves away from it at a steady pace.
- The graph is never parallel to the *vertical asymptote*, but moves toward it at a steady pace.
- The graph has a distinct concavity, which, depending on a transformation, can be concave up or down.
- There is always an x-intercept.
- There is at most one y-intercept. This means that some graphs may have no y-intercept, while others may have one.

Problem 1:

Find the following for $h(x) = \log x$.

a. Domain
b. Coordinates of the x-intercept
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$x > 0$

Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept:

$0 = \log x$

and changing to exponential form, we find

$x = 10^0$

$x = 1$

The coordinates are $(1, 0)$.

- Coordinates of the y-intercept:

  $h(0) = \log 0$

  But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

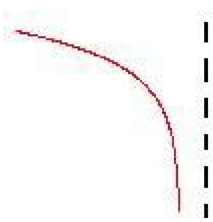
  Since our equation is of the form $y = \log_b x$ with $b = 10$, the y-axis is the vertical asymptote whose equation is $x = 0$.

Problem 2:

Find the following for $f(x) = \log(-x)$.

a. Domain
b. Coordinates of the x-intercept
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

  The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

  We do this by finding the numbers that make the argument greater than 0. That is,

  $-x > 0$

  Multiplying both sides by -1, we find

  $x < 0$

  Therefore, in *Interval Notation*, the domain is $(-\infty, 0)$.

- Coordinates of the x-intercept:

  $0 = \log(-x)$

  $10^0 = -x$

  $-x = 1$

  $x = -1$

  The coordinates are $(-1, 0)$.

- Coordinates of the y-intercept:

  $f(0) = \log 0$

  But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

  This is a reflection of $y = \log x$ about the y-axis. Horizontal shifts of $y = \log_b x$ affect the location of the *vertical asymptote.* Reflections about the y-axis and horizontal stretches/shrinks affect the location of the *vertical asymptote* indirectly because they affect the horizontal shift.

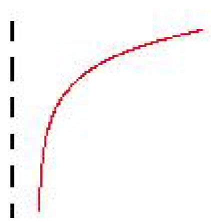
  Since we have no horizontal shift, the reflection about the y-axis does not affect the equation of the vertical asymptote, which is still $x = 0$.

Problem 3:

Find the following for $f(x) = \log(x+2) - 1$.

a. Domain
b. Coordinates of the x-intercept
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

  The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$x + 2 > 0$

$x > -2$

Therefore, in *Interval Notation*, the domain is $(-2, \infty)$.

- Coordinates of the x-intercept:

$$0 = \log(x+2) - 1$$

$$\log(x+2) = 1$$
$$x + 2 = 10^1$$
$$x = 8$$

The coordinates are $(8, 0)$.

- Coordinates of the y-intercept rounded to 2 decimal places:

$$f(0) = \log(0+2) - 1 = \log 2 - 1 \approx -0.6990$$

The coordinates are $(0, -0.70)$.

- Equation of the Vertical Asymptote:

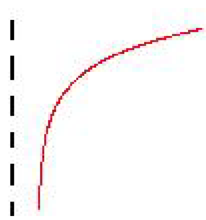
This is a horizontal shift of $y = \log x$ by 2 units to the left and a vertical shift of 1 unit down. Since horizontal shifts affect the location of the vertical asymptote, and we do have a horizontal shift 2 units to the left, the equation of the vertical asymptote becomes $x = -2$. Vertical shifts **DO NOT** affect it.

Problem 4:

Find the following for $k(x) = \log x + 2$.

a. Domain
b. Coordinates of the x-intercept. Round to 2 decimal places.
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

  The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

  We do this by finding the numbers that make the argument greater than 0. That is,

  $$x > 0$$

  Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept rounded to 2 decimal places:

  $$0 = \log x + 2$$

  $$\log x = -2$$
  $$x = 10^{-2} = 0.01$$

  The coordinates are $(0.01, 0)$.

- Coordinates of the y-intercept:

  $$k(0) = \log 0 + 2$$

  But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

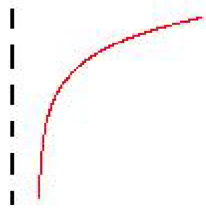
  This is a vertical shift of $y = \log x$ by 2 units upward. Vertical shifts **DO NOT** affect the location of the vertical asymptote. Therefore, the equation of the vertical asymptote is still $x = 0$.

Problem 5:

Find the following for $g(x) = \ln x$. Note we a discussing the natural logarithm "el en of x"!

a. Domain
b. Coordinates of the x-intercept
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

  The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

  We do this by finding the numbers that make the argument greater than 0. That is,

  $x > 0$

  Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept:

  $0 = \ln x$

  $e^0 = x$

  $x = 1$

  The coordinates are $(1, 0)$.

- Coordinates of the y-intercept:

  $g(0) = \ln 0$

  But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

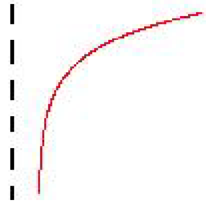
  Since our equation is of the form $y = \log_b x$ with $b = e$, the y-axis is the vertical asymptote whose equation is $x = 0$.

Problem 6:

Find the following for $p(x) = \log_2(x-1) + 3$.

a. Domain
b. Coordinates of the x-intercept
c. Coordinates of the y-intercept
d. Equation of the vertical asymptote

The graph has the following shape:



- Domain:

  The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

  We do this by finding the numbers that make the argument greater than 0. That is,

  $x - 1 > 0$

  $x > 1$

  Therefore, in *Interval Notation*, the domain is $(1, \infty)$.

- Coordinates of the x-intercept:

  $$0 = \log_2(x-1) + 3$$

  $$\log_2(x-1) = -3$$
  $$x - 1 = 2^{-3}$$
  $$x = \tfrac{1}{8} + 1 = 1\tfrac{1}{8} = \tfrac{9}{8}$$

  The coordinates are $(\tfrac{9}{8}, 0)$.

- Coordinates of the y-intercept:

  $$p(0) = \log_2(0-1) + 3 = \log_2(-1) + 3$$

  But any logarithm of a negative number is imaginary. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

  This is a horizontal shift of $y = \log_2 x$ by 1 unit to the right and a vertical shift of 3 units up. Since horizontal shifts affect the location of the vertical asymptote, and we do have a horizontal shift 1 unit to the right, the equation of the *vertical asymptote* becomes $x = 1$. Vertical shifts **DO NOT** affect the location of the vertical asymptote.