DETAILED SOLUTIONS AND CONCEPTS - DECIMALS AND WHOLE NUMBERS

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ARITHMETIC TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

ADDITION

Vocabulary:

In ***8 + 2 = 10***, the ***10*** is called **Sum.**

Rule for Adding Whole Numbers:

- Align the numbers in columns by place value.
- Then add the digits in each column starting on the right.
- Regroup whenever the sum of a column is more than one digit.

Problem 1:

Find the sum of ***180 + 87 + 15*** without a calculator.

$$\begin{array}{r} 180 \\ 87 \\ +\ 15 \\ 11 \\ \hline 282 \end{array}$$

The sum of the digits in the *ones* column is ***12***. Record the ***2*** in the *ones* column of the sum and regroup the ***1*** to the *tens* column.

Now the sum of the digits in the *tens* column of ***18***. Record the ***8*** in the *tens* column of the sum and regroup another ***1*** to the *hundreds* column.

The sum of the digits in the *hundreds* column is ***2***. Record it in the *hundreds* column of the sum.

We find the sum to be *282*.

Rule for Adding Decimals:

- Align the numbers in columns by place value with the decimal points directly under each other.
- Then add the digits of each column starting on the right.
- Regroup whenever the sum of a column is more than one digit.
- Place the decimal point in the sum directly under the other decimal points.

Problem 2:

Find the sum of ***67.9 + 23 + 0.34*** without a calculator.

$$\begin{array}{r} 67.90 \\ 23.00 \\ +\ 0.34 \\ 11 \\ \hline 91.24 \end{array}$$

The decimal point in ***23*** is understood to be at the right end.

It is preferable to write the amounts so that all have the same number of decimal places by attaching ending zeros.

We find the sum to be *91.24*.

SUBTRACTION

Vocabulary:

In ***8 − 2 = 6***, the ***8*** is called **Minuend**, the ***2*** is the **Subtrahend**, and ***6*** is called the **Difference**.

Rule for Subtracting Whole Numbers:

- Align the two numbers in columns by place value.
- Then subtract the digits in each column starting on the right.
- Regroup whenever subtracting a larger digit from a smaller one.

Problem 3: (An alternate way of subtracting will be shown!)

Find the difference of ***8,034 − 5,679*** without a calculator.

$$\begin{array}{r} 8034 \\ -5679 \\ 111 \\ \hline 2355 \end{array}$$

In the *ones* column ***9*** is more than ***4***. We change the ***4*** to ***14*** by regrouping the ***1*** to the *tens* column. ***9*** subtracted from ***14*** equals ***5*** which we record in the *ones* column of the difference.

In the *tens* column, we first add ***1*** to ***7***, but then ***8*** is more than ***3***. We change the ***3*** to ***13*** by regrouping the ***1*** to the *hundreds* column. ***8*** subtracted from ***13*** equals ***5*** which we record in the *tens* column of the difference.

In the *hundreds* column, we first add ***1*** to ***6***, but ***7*** is more than ***0***. We change the ***0*** to ***10*** by regrouping the ***1*** to the *thousands* column. ***7*** subtracted from ***10*** equals ***3*** which we record in the *hundreds* column of the difference.

In the *thousands* column, we first add ***1*** to ***5***, and since ***5*** is less than ***8***, we subtract and record ***2*** in the *thousands* column of the difference.

We find the difference to be *2,355*.

Rule for Subtracting Decimals:

- Align the numbers in columns by place value with the decimal points directly under each other.
- Then subtract the digits in each column starting on the right.
- Regroup whenever subtracting a larger digit from a smaller one.
- Place the decimal point in the difference directly under the other decimal points.

Problem 4: (Again, an alternate way of subtracting will be shown!)

Find the difference of $201 - 72.35$ without a calculator.

$$\begin{array}{r} 201.00 \\ -\ 72.35 \\ 111\ 1 \\ \hline 128.65 \end{array}$$

The decimal point in 201 is understood to be at the right end.

It is preferable to write the amounts so that all have the same number of decimal places by attaching ending zeros.

We find the difference to be 128.65.

Calculator Tip: When using the calculator to find the difference of these numbers you do not have to attach zeros to the end.

MULTIPLICATION

Vocabulary:

In $8 \cdot 2 = 16$, the 8 is called **Multiplicand**, the 2 is the **Multiplier**, and 16 is called the **Product**.

NOTE:

Given that $16 = 8 \cdot 2$, we can say that 8 and 2 are **factors** of 16! That is, when a number is the product of two or more numbers, each of the latter is called a **factor** of the former.

Various Notations for Multiplication:

$8 \cdot 2 \quad 8 * 2 \quad 8 \times 2 \quad 8(2)$

In higher mathematics parentheses ( ) are most often used to indicate multiplication!

Rule for Multiplying Whole Numbers:

- Place the two numbers one under the other.
- Multiply the multiplicand, in turn, by each digit of the multiplier starting with the ones place.
- Align the ones digit of each of these "partial products" with their multiplier digit.
- Add the "partial products" as they are aligned

Problem 5:

Find the product of ***528*** × ***203*** without a calculator.

```
     528
    ×203
  ------
    1584
    000
 + 1056
  ------
  107184
```

Multiply ***3(528)*** and align the product (1584) under the 3 of the multiplier.
Multiply ***0(528)*** and align the product (0) under the 3 of the multiplier.
Multiply ***2(528)*** and align the product (1056) under the 3 of the multiplier.

Add the "partial products" in their current alignment assuming zeros in empty spaces.

We find the product to be *107,184*.

Problem 6:

Find the product of ***567*** × ***1000*** without a calculator.

A quick way to multiply a whole number by a multiple of 10 without a calculator is to move the decimal point in the number to the RIGHT as many places as the number of zeros. Attach zeros if necessary.

There are 3 zeros in 1000, therefore we move the decimal point 3 places to the RIGHT!

567* × *1000 = 567000

NOTE: We had to attach three zeros!

Rule for Multiplying Decimals:

- Place the two numbers one under the other.
- Ignore the decimal point and multiply just like did with whole numbers.
- Count the number of decimal places in the multiplicand and in the multiplier.
- In the product, count from the right the number of digits equal to the sum of the decimal places of the multiplicand and the multiplier. This is where the decimal point will be placed.

Problem 7:

Find the product of ***0.291*** × ***0.14*** without a calculator.

$$\begin{array}{r} 0.291 \\ \times\ 0.14 \\ \hline 1164 \\ +\ 291 \\ \hline 0.04074 \end{array}$$

Three (3) decimal places plus two (2) decimal places equals five (5) decimal places.

There are NO decimal points in the "partial products."

There were not enough digits in the final product to accommodated five (5) decimal places. Therefore, we had to insert a zero to the left of the product!

We find the product to be *0.04074*.

Problem 8:

Find the product of ***24.5*** × ***100*** without a calculator.

NOTE: A quick way to multiply a decimal by a multiple of 10 without a calculator is to move the decimal point in the number to the RIGHT as many places as the number of zeros. Attach zeros if necessary.

There are 2 zeros in 100, therefore we move the decimal point 2 places to the RIGHT!

Here we move the decimal point two places to the right!

24.5 × ***100 = 2450***

NOTE: We had to attach one zero!

DIVISION

Vocabulary:

In ***8*** ÷ ***2 = 4***, the ***8*** is called **Dividend**, the ***2*** is the **Divisor**, and ***4*** is called the **Quotient**.

NOTE:

16 ÷ ***5*** has a quotient that consists of the whole number ***3*** and a **Remainder** of ***1***. The remainder is the part left over after long division.

Various Notations for Division:

$8 \div 2 \quad 2\overline{)8} \quad \frac{8}{2} \quad {}^{8}\!/_{2} \quad 8 / 2$

Rule:

Instead of creating a rule, the division of whole numbers and decimals will be illustrated by using examples!

Problem 9:

Find the quotient of $7\overline{)2135}$ without a calculator.

Starting on the left, find the first group of digits of the dividend that is larger or equal to the divisor. This results in the number ***21***.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \end{array}$$

Divide ***21*** by ***7*** and write ***3*** above the rightmost digit of ***21***.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ 21 \end{array}$$

Multiply ***3*** by ***7*** and write this product below the digits ***21*** of the dividend. Align places!

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ -\underline{21} \\ 0 \end{array}$$

Subtract the product from ***21*** and write the difference ***0*** below the product aligning places.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ -\underline{21} \\ 03 \end{array}$$

To the right of the difference ***0*** write the next digit of the dividend, which is ***3***.

$$\begin{array}{r} 30 \\ 7\overline{)2135} \\ -\underline{21} \\ 03 \\ -\underline{0} \\ 3 \end{array}$$

Please note that ***7*** divides into ***3*** zero times! That's why the next digit of the quotient is ***0***.

$$\begin{array}{r} 30 \\ 7\overline{)\,2135} \\ -21 \\ \hline 03 \\ -0 \\ \hline 35 \end{array}$$

To the right of the difference we write the next digit of the dividend, which is ***5***.

Finally,

$$\begin{array}{r} 305 \\ 7\overline{)\,2135} \\ -21 \\ \hline 03 \\ -0 \\ \hline 35 \\ -35 \\ \hline 0 \end{array}$$

We find the quotient to be *305*. There is NO remainder.

Problem 10:

Find the quotient of $8\overline{)5.6}$ without a calculator.

Since the dividend is a decimal number, our first task is to insert a decimal point into the quotient. It must be placed right above the decimal point of the dividend.

$$\begin{array}{r} . \\ 8\overline{)5.6} \end{array}$$

Since ***8*** is larger than the whole number ***5*** of the dividend, the whole number part of the quotient will be ***0***.

$$\begin{array}{r} 0. \\ 8\overline{)5.6} \end{array}$$

To continue the division process, we now have to use a digit from the fractional part of the dividend. We will ignore the decimal point in the dividend and divide ***8*** into ***56***. However, all digits of the quotient MUST now be placed to the right of the decimal point.

$$\begin{array}{r} 0.7 \\ 8\overline{)\,5.6} \\ -5\,6 \\ \hline 0 \end{array}$$

We find the quotient to be *0.7*. There is NO remainder.

Problem 11:

Find the quotient of $2.3\overline{)3.68}$ without a calculator.

NOTE: The divisor must always be changed to a whole number!

Since the divisor is NOT a whole number, the first thing we will do is to move its decimal point so that it is on the right side of all digits.

Next, we'll move the decimal point in the dividend to the right as many places as the decimal point was moved in the divisor. Attach ending zeros if necessary!

$$23\overline{)36.8}$$

Since the divisor is now a whole number, we'll insert a decimal point into the quotient. Remember that it must be placed right above the decimal point of the dividend.

$$\begin{array}{r} . \\ 23\overline{)36.8} \end{array}$$

Since ***23*** is larger than the whole number ***36*** of the dividend and therefore divides into it (once), the whole number part of the quotient will be ***1***.

$$\begin{array}{r} 1. \\ 23\overline{)36.8} \\ -23 \\ \hline 13 \end{array}$$

To continue the division process, we now have to use a digit from the fractional part of the dividend. We will ignore the decimal point in the dividend and divide ***23*** into ***138***. However, all digits of the quotient MUST now be placed to the right of the decimal point.

$$\begin{array}{r} 1.6 \\ 23\overline{)\ 36.8} \\ -\ 23 \\ \hline 13\ 8 \\ -\ 13\ 8 \\ \hline 0 \end{array}$$

We find the quotient to be *1.6*. There is NO remainder.

Problem 12:

Find the quotient of ***567 ÷ 10000*** without a calculator.

NOTE: A quick way to divide a number by a multiple of 10 without a calculator is to move the decimal point in the number to the LEFT as many places as the number of zeros. Insert zeros if necessary.

Here we move the decimal point four places to the left!

567 ÷ 10000 = 0.0567 Here we had to insert one zero.

Problem 13:

Find the quotient of ***786580 ÷ 100*** without a calculator.

Here we move the decimal point two places to the left!

786580 ÷ 100 = 7865.8

Problem 14:

Find the quotient of ***198.78 ÷ 10***.

Here we move the decimal point one place to the left!

198.78 ÷ 10 = 19.878

ROUNDING OF A DECIMAL NUMBER

Rounding a decimal number to a certain place value means that we want to find an **approximation** of the number. Rounding is the process for anyone who doesn't care to be exact. That's okay to a point because there are many different uses for numbers. Generally, the size of the number and its use dictates the place to which it should be rounded.

For example, the value for the number π ***(pi)*** given by the TI 30X IIS/B calculator is ***3.141592954***. However, we often use a less accurate, but still useful value of 3.14.

In this case, we would say that $\pi \approx$ ***3.14***. This is read as "pi is approximately equal to 3.14."

When NASA sends the shuttle to the space station it might want to use the highly accurate value of the number π to ensure successful docking, but a carpenter cutting a circular arch is probably satisfied with a value of 3.14.

Rule:

- Locate the digit that occupies the rounding place.
- If the digit to the right of the rounding place is **less than 5**, leave the digit in the rounding place unchanged. Given a whole number, fill the remaining places with zeros. Given a decimal number, drop the remaining decimal places.
- If the digit to the right of the rounding place is **greater than 5 or equal to 5**, add ***1*** to the digit in the rounding place. This is called "rounding up." Given a whole number, fill the remaining places with zeros. Given a decimal number, drop the remaining decimal places.

Problem 15:

Round ***6,296*** to the nearest thousands place.

6 2 9 6

6 is in the thousands place. The digit to its right is 2, which is less than 5.

Therefore, ***6,296*** $\approx$ ***6,000***. Note that the remaining places were filled with zeros!

Problem 16:

Round ***96.7945*** to the nearest hundredths place!

9 6 . 7 9 4 5

9 is in the second decimal place. The digit to its right is 4, which is less than 5.

Therefore, ***96.7945*** $\approx$ ***96.79***. Note that the remaining places were dropped!

Problem 17:

Round ***5,371*** to the nearest hundreds place.

5 3 7 1

3 is in the hundreds place. The digit to its right is 7, which is greater than 5.

Therefore, ***5,371*** ≈ ***5,400***. Note that the remaining places were filled with zeros!

Problem 18:

Round ***1,795*** to the nearest tens place.

1 7 9 5

9 is in the tens place. The digit to its right is 5.

Note: When the digit in the rounding place is 9 and must be rounded up it becomes 10. The 0 replaces the 9 and the 1 is regrouped to the next place to the left.

Therefore, ***1,795*** ≈ ***1,800***. Note that the remaining places were filled with zeros!

Problem 19:

Round ***58.6854*** to the nearest tenths place!

5 8 . 6 8 5 4

6 is in the first decimal place. The digit to its right is 8, which is greater than 5.

Therefore, ***58.6854*** ≈ ***58.7***. Note that the remaining places were dropped!

Problem 20:

Round ***21.1974*** to the nearest hundredths place!

2 1 . 1 9 7 4

9 is in the second decimal place. The digit to its right is 7, which is greater than 5.

Note: When the digit in the rounding place is 9 and must be rounded up it becomes 10. The 0 replaces the 9 and the 1 is regrouped to the next place to the left.

Therefore, ***21.1974*** ≈ ***21.20***. Note that the remaining places were dropped! The ***0*** in the hundredths place has to show up to indicate that we rounded to two decimal places.

Problem 21:

Estimate the sum of ***0.935 + 12.54 + 152.07 + 18*** by rounding to tens. Then find the exact sum.

To estimate the solution to an addition problem we do the following:

- **Round each number to a specific place value or to a number with one nonzero digit.**
- **Add the rounded numbers.**

Instead of ***0.935 + 12.54 + 152.07 + 18*** we think ***0 + 10 + 150 + 20 = 180***.

The exact sum is 183.545.

Problem 22:

Estimate the sum of ***24,003 + 5,874 + 319,467 + 52,855*** by rounding to thousands. Then find the exact sum.

Instead of ***24,003 + 5,874 + 319,467 + 52,855*** we think ***24,000 + 6,000 + 319,000 + 53,000 = 402,000***.

The exact sum is 402,199.

Problem 23:

Estimate the difference of ***427.45 - 125*** by rounding to hundreds. Then find the exact sum.

To estimate the solution to a subtraction problem we do the following:

- **Round each number to a specific place value or to a number with one nonzero digit.**
- **Subtract the rounded numbers.**

Instead of ***427.45 - 125*** we think ***400 - 100 = 300***.

The exact difference is 302.45.

Problem 24:

Estimate the difference of ***4,048 - 36*** by rounding to tens. Then find the exact sum.

Instead of ***4,048 - 38*** we think ***4,050 - 40 = 4,010***.

The exact difference is 4,012.

Problem 25:

Estimate the cost of 38 light bulbs if each bulb costs $1.15 by rounding to one nonzero digit. Then find the exact price.

To estimate the solution to a multiplication problem we do the following:

- **Round each number to a specific place value or to a number with one nonzero digit.**
- **Multiply the rounded numbers.**

Instead of 38×1.15 we think $40 \times 1 = 40$.

The exact price is $43.70.

Problem 26:

Estimate the cost of 54 knobs for your new kitchen cabinets if each know costs $3.40 by rounding to one nonzero digit. Then find the exact price.

Instead of 54×3.40 we think $50 \times 3 = 150$.

The exact price is $183.60. In this case, the difference between the estimate and the exact product is quite large. This is due to the fact that both values were on the border of having to be rounded up.

Problem 27:

The owner of a sandwich shop tells you that he will charge you $108 for 18 sandwiches that you want to buy for your party. Estimate the cost of one sandwich by rounding to one nonzero digit. Then find the exact price.

To estimate the solution to a division problem containing only whole numbers we do the following:

- **Round each number to a specific place value or to a number with one nonzero digit.**
- **Divide the rounded numbers.**

Instead of $108 \div 18$ we think $100 \div 20 = 5$.

The exact price for one sandwich is $6.

Problem 28:

The florist tells you that she will charge you $915 for 25 flower baskets that you want to place on each table at your wedding reception. Estimate the cost of one basket by rounding to one nonzero digit. Then find the exact price.

Instead of $915 \div 25$ we think $900 \div 30 = 30$.

The exact price for one flower basket is $36.60.

Problem 29:

Which of the following numbers is the smallest?

0.016 0.106 0.16 0.601

To compare decimal numbers we do the following:

- **Arrange the numbers vertically and add zeros, if necessary, to the end of numbers so that all numbers have the same number of digits.**
- **Compare the whole numbers.**
- **If the whole numbers are equal, compare the numbers in the decimal places staring with the first decimal place.**
- **When two digits in the same decimal place are different then the digit that is larger/smaller determines the larger/smaller decimal number.**

Let's arrange the numbers vertically and add zeros to the end of some numbers so that all numbers have three digits.

0.016
0.106
0.160
0.601

The whole number parts are all zero. Therefore, we'll move to the first decimal place. When comparing the digits we find that the first number has the smallest one. Therefore, 0.016 is the smallest number given our choices.

Problem 30:

Which of the following numbers is the smallest?

0.097 0.3 0.103 0.023

Again, let's arrange the number vertically and add zeros to the end of some numbers so that all numbers have three digits.

0.097
0.300
0.103
0.023

The whole number parts are all zero. Therefore, we'll move to the first decimal place. When comparing the digits we find that the first number and the last number have the the smallest one. Therefore, we have to compare the digits in the second decimal place. Here we find that the digit of the last number is smaller than the digit of the first number. Therefore, 0.023 is the smallest number given our choices.

Problem 31:

A painter worked 3 hours on Monday and 3.5 times as many hours on Tuesday. Which of the following ways should be used to calculate how many hours the painter worked on both days?

$3 + 3.5$ $3 \times (3.5 + 3)$ $3 - 3.5$ $3 + (3.5 \times 3)$

The answer is $3 + (3.5 \times 3)$ because the phrase "3.5 times as many hours (Monday)" always indicates multiplication.

Problem 32:

A waiter worked 8 hours on Friday and 1.5 times as many hours on Saturday. Which of the following ways should be used to calculate how many hours the waiter worked on both days?

$8 + (1.5 \times 8)$ $8 + 1.5$ $8 - (1.5 + 8)$ 8×1.5

The answer is $8 + (1.5 \times 8)$ because the phrase "1.5 times as many hours (Friday)" again indicates multiplication.

Problem 33:

On four tests, a student had the grades of 78, 89, 45, and 80. What is her average grade?

To find the average of a group of numbers you must do the following:

- **Add the numbers.**
- **Divide the sum by the number of addends.**

NOTE: The Average is also often called the MEAN!

In our case, let's find the sum $78 + 89 + 45 + 80 = 292$

Since we have 4 numbers (addends) we then divide 292 by 4. This equals 73.

Therefore, the student's average grade is a 73.