



DETAILED SOLUTIONS AND CONCEPTS - EQUATIONS WITH RATIONAL EXPONENTS

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Finding Solutions of Equations with Rational Exponents

You can encounter infinitely many different equations with rational exponents. Some of them can only be solved using extremely complex procedures. However, the equations reducible to the form $u^{m/n} = d$ can be solved by the method below.

Let u be an algebraic expression, d a constant, m and n positive integers, and the fractional exponent $\frac{m}{n}$ in lowest terms.

Solve $u^{m/n} = d$ as follows:

If the numerator m in the fractional exponent is even, then

$$(u^{m/n})^{n/m} = \pm d^{n/m} \text{ and } u = \pm d^{n/m}$$

If the numerator m in the fractional exponent is odd, then

$$(u^{m/n})^{n/m} = d^{n/m} \text{ and } u = d^{n/m}$$

- Isolate the term raised to the rational power on one side of the equation. Be sure the coefficient is a **positive 1**.
- Raise both sides of the equation to a power that is the reciprocal of the power on the variable.
- If necessary, further isolate the variable.
- You might have to use your calculator to get a decimal equivalent of the solution.
- Any time we raise both sides of an equation to an even power we **MUST** check our solutions in the original equation rejecting any that do not satisfy it.

Problem 1:

$$\text{Solve } x^{1/3} + 1 = 0.$$

We have to isolate the term raised to the power

$$x^{1/3} = -1$$

Since the numerator of the rational power is odd, we get

$$(x^{1/3})^3 = (-1)^3$$

$$x = -1$$

Problem 2:

$$\text{Solve } 3x^{4/5} - 6 = 0 \text{ and round your answer to three decimal places.}$$

We have to isolate the term raised to the power

$$3x^{4/5} = 6$$

$$x^{4/5} = 2$$

$$x \approx \pm 2.378$$

Since the numerator of the rational power is even, we get

$$(x^{4/5})^{5/4} = \pm(2)^{5/4} = \pm(\sqrt[4]{2})^5$$

In this case, we'll use the calculator to find

$$x = -2.378 \text{ or } x = 2.378$$

Problem 3:

$$\text{Solve } \sqrt{5 - 4x} - 2 = 0. \text{ Find real solutions only!}$$

Remember that radicals can be written as exponents as follows

$$(5 - 4x)^{1/2} - 2 = 0$$

We have to isolate the term raised to the power

$$(5 - 4x)^{1/2} = 2$$

Since the numerator of the rational power is odd, we get

$$[(5 - 4x)^{1/2}]^2 = 2^2$$

$$5 - 4x = 4$$

$$-4x = -1$$

and dividing both sides by **-4**, we get

$$x = \frac{1}{4}$$

Any time we raise both sides of an equation to an even power we **MUST** check our answers in the original equation.

In this case, we must replace the variable with $\frac{1}{4}$ in the original equation to make sure that the value to the left of the equal sign becomes equal to the one on the right, which is **0**.

In our case we find that

$$\sqrt{5 - 4\left(\frac{1}{4}\right)} - 2 = \sqrt{4} - 2 = 2 - 2 = 0$$

Since the value of the left side equals 0, which is the value of the right side, we can say that the solution is indeed $\frac{1}{4}$.

Problem 4:

Solve $\sqrt{3 - x} + 1 = 0$. Find real solutions only!

Let's isolate the term raised to the power

$$\sqrt{3 - x} = -1$$

Please note that a square root is never equal to a negative number. There is actually NO solution to this problem. However, let's just go ahead and pretend that we did not notice.

Since the numerator of the rational power is odd we get

$$(\sqrt{3 - x})^2 = (-1)^2$$

$$3 - x = 1$$

$$x = 2$$

Any time we raise both sides of an equation to an even power we **MUST** check our answers in the original equation.

In this case, we must replace the variable with **2** in the original equation to make sure that the value to the left of the equal sign becomes equal to the one on the right, which is **0**.

$$\sqrt{3-2} + 1 = \sqrt{1} + 1 = 1 + 1 = 2$$

Since the value of the left side is 2, we can say that **2** is NOT a solution because it is not equal to the value of the right side, which is 0.

This equation has NO solutions.

Problem 5:

$$\text{Solve } (x-3)^{2/3} - 9 = 0$$

We have to isolate the term raised to the power

$$(x-3)^{2/3} = 9$$

Since the numerator of the rational power is even, we get

$$[(x-3)^{2/3}]^{3/2} = \pm(9)^{3/2} = \pm(\sqrt{9})^3 = \pm 27$$

$$x-3 = \pm 27$$

and further isolating the variable, we get

$$x = \pm 27 + 3$$

$$x = 27 + 3 = 30 \text{ or } x = -27 + 3 = -24$$

Problem 6:

$$\text{Solve } (x-3)^{2/3} = 0$$

Please note that the numerator of the rational power is even, however, $\pm 0 = 0$.

$$[(x-3)^{2/3}]^{3/2} = (0)^{3/2}$$

$$x-3 = 0$$

$$x = 3$$