



## DETAILED SOLUTIONS AND CONCEPTS - QUADRATIC EQUATIONS

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Thank you!

**PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!**

### Quadratic Equations

A quadratic equation in  $x$  is an equation that can be written in the standard form

$ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers, but  $a \neq 0$ .

### Solving Quadratic Equations

#### A. Factoring Method - Some, but not all quadratic equations can be solved by factoring.

- Write the quadratic equation in standard form. That is, using *Properties of Equality*, bring all terms to one side of the equation so that the other side is equal to  $0$ . Combine like terms, if possible.
- Factor relative to the integers.
- Use the *Zero Product Principle* to solve. It states that if a product is equal to zero, then at least one of the factors is equal to zero.

#### Problem 1:

Solve  $15x^2 - 5x = 0$  using the *Factoring Method*. Find any real and imaginary solutions.

The quadratic equation is already in standard form so that we can factor as follows:

$$5x(3x - 1) = 0$$

By the *Zero Product Principle* we can say

$$5x = 0 \text{ or } 3x - 1 = 0$$

Therefore,  $x = 0$  or  $x = \frac{1}{3}$ .

Check:

Please note that if we check these solutions in the original equation, we find the following:

Given  $15x^2 - 5x = 0$  and using  $x = 0$ , is  $15(0)^2 - 5(0)$  equal to  $0$ ? Yes! Therefore the solution is correct!

Given  $15x^2 - 5x = 0$  and using  $x = \frac{1}{3}$ , is  $15(\frac{1}{3})^2 - 5(\frac{1}{3})$  equal to  $0$ ? Yes, because  $\frac{15}{9} - \frac{5}{3} = \frac{15}{9} - \frac{15}{9}$ ! Therefore the solution is correct!

### Problem 2:

Solve  $x^2 + 5x = -6$  using the *Factoring Method*. Find any real and imaginary solutions.

Rewriting the equation in standard form yields  $x^2 + 5x + 6 = 0$ .

We notice that we have a trinomial, which we can factor as follows:

$$(x + 3)(x + 2) = 0$$

By the *Zero Product Principle* we can say

$$x + 3 = 0 \text{ or } x + 2 = 0$$

Therefore,  $x = -3$  or  $x = -2$ .

Check:

Please note that if we check these solutions in the original equation, we find the following:

Given  $x^2 + 5x = -6$  and using  $x = -3$ , is  $(-3)^2 + 5(-3)$  equal to  $-6$ ? Yes! Therefore the solution is correct!

Given  $x^2 + 5x = -6$  and using  $x = -2$ , is  $(-2)^2 + 5(-2)$  equal to  $-6$ ? Yes! Therefore the solution is correct!

### Problem 3:

Solve  $x^2 - 4x + 4 = 0$  using the *Factoring Method*. Find any real and imaginary solutions.

We notice that we have a trinomial, which we can factor as follows:

$$(x - 2)(x - 2) = 0$$

By the *Zero Product Principle* we can say

$$x - 2 = 0 \text{ or } x - 2 = 0$$

Therefore,  $x = 2$  in both cases. Sometimes the  $2$  is considered to be a **double root**.

**B. Square Root Method** - All quadratic equations can be solved by this method which uses the *Square Root Property*. However, we might want to avoid this method for some quadratic equations because it can get very cumbersome.

- Isolate the squared term on one side of the equation. Be sure its coefficient is a **positive 1** !!!!!
- Apply the *Square Root Property*.
- If necessary, further isolate the variable.

### The Square Root Property

If  $u$  is an algebraic expression containing a variable and  $d$  is a constant, then  $u^2 = d$  has exactly two solutions, namely

$$u = \sqrt{d} \text{ and } u = -\sqrt{d} \text{ or simply } u = \pm\sqrt{d}.$$

#### Problem 4:

Solve  $x^2 - 16 = 0$  using the *Square Root Method*. Find any real and imaginary solutions.

Isolate the squared term

$$x^2 = 16$$

and use the *Square Root Property*

$$x = \pm\sqrt{16}$$

then  $x = 4$  or  $x = -4$ .

#### Problem 5:

Solve  $x^2 - 5 = 0$  using the *Square Root Method*. Find any real and imaginary solutions.

Isolate the squared term

$$x^2 = 5$$

and use the *Square Root Property*

$$x = \pm\sqrt{5}$$

Therefore,  $x = \sqrt{5}$  or  $x = -\sqrt{5}$ .

### Problem 6:

Solve  $(x - 2)^2 + 8 = 0$  using the *Square Root Method*. Find any real and imaginary solutions.

Isolate the squared term

$$(x - 2)^2 = -8$$

use the *Square Root Property*

$$x - 2 = \pm\sqrt{-8}$$

Please observe that we found an imaginary number. This means we have an imaginary solution.

We can further isolate the variable

$$x = \pm i\sqrt{8} + 2$$

and lastly simplify the square root as follows:

$$x = \pm 2i\sqrt{2} + 2$$

Therefore,  $x = 2i\sqrt{2} + 2$  or  $x = -2i\sqrt{2} + 2$ .

**C. Quadratic Formula Method** - This method can be used to solve any type of quadratic equation. The *Quadratic Formula* was derived by first applying the *Square Completion Method* to the general form of the quadratic equation  $ax^2 + bx + c = 0$  and then using the *Square Root Method* to solve for  $x$ .

#### The Quadratic Formula

The solutions of a quadratic equation in general form  $ax^2 + bx + c = 0$  with  $a \neq 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Write the quadratic equation in standard form. That is, using properties of equality, bring all terms to one side of the equation so that the other side is equal to  $0$ . Combine like terms, if possible.
- $a$  is the coefficient of  $x^2$ ,  $b$  is the coefficient of  $x$ , and  $c$  is the constant.

Note:  $b$  and/or  $c$  can be zero or a negative number!!!  $a$  cannot be equal to zero, but it can be negative.

- Substitute the values of **a**, **b**, and **c** into the quadratic formula. Be sure to note that a negative value for **b** in the quadratic equation does not replace the negative sign in front of **b** in the formula.
- Perform all arithmetic operations to find simplified solutions, such as adding like terms, simplifying square roots, and reducing fractions.

### Problem 7:

Solve  $x^2 - 10x + 20 = 0$  using the *Quadratic Formula Method*. Find any real and imaginary solutions.

For the *Quadratic Formula*, we need the the values **a = 1**, **b = -10**, and **c = 20** from the given equation. Then,

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)}$$

Next, we combine like terms and simplify the radical

$$x = \frac{10 \pm \sqrt{20}}{2} = \frac{10 \pm 2\sqrt{5}}{2}$$

and finally, we reduce the answer to lowest term

$$x = \frac{2(5 \pm \sqrt{5})}{2} = 5 \pm \sqrt{5}$$

Therefore,  $x = 5 + \sqrt{5}$  or  $x = 5 - \sqrt{5}$ .

### Problem 8:

Solve  $4x^2 - 8x + 11 = 0$  using the *Quadratic Formula Method*. Find any real and imaginary solutions.

In this case, **a = 4** **b = -8** **c = 11**

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(11)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 - 176}}{8}$$

$$x = \frac{8 \pm \sqrt{-112}}{8}$$

Please observe that we found an imaginary number. This means we have an imaginary solution.

We could further simplify to get

$$x = \frac{8 \pm 4i\sqrt{7}}{8}$$

and finally, we will reduce the fraction to find

$$x = \frac{2 \pm i\sqrt{7}}{2}$$

Therefore,  $x = \frac{2 + i\sqrt{7}}{2}$  or  $x = \frac{2 - i\sqrt{7}}{2}$ .

**Please note that we could NOT have solved this quadratic equation by the Factoring Method.**

### Problem 9:

Solve  $x^2 - 9 = 0$  using the *Quadratic Formula*, *Factoring*, and the *Square Root Property*. Find any real and imaginary solutions.

#### a. Using the Quadratic Formula

In this case,  $a = 1$ ,  $b = 0$ , and  $c = -9$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{\pm \sqrt{36}}{2}$$

$$x = \pm 3$$

Therefore,  $x = -3$  or  $x = 3$ .

**Please note that we could have solved this quadratic equation also by using the Factoring or Square Root Methods.**

## b. Using Factoring and the Zero Product Principle

Solve  $x^2 - 9 = 0$ .

Notice that they are dealing with a *Difference of Squares*  $x^2 - a^2 = (x - a)(x + a)$ , where  $a = 3$ .

Therefore, we can factor as follows:

$$(x - 3)(x + 3) = 0$$

Therefore,  $x = 3$  or  $x = -3$ .

Check:

Please note that if we check these solutions in the original equation, we find the following:

Given  $x^2 - 9 = 0$  and using  $x = 3$ , is  $(3)^2 - 9$  equal to  $0$ ? Yes! Therefore the solution is correct!

Given  $x^2 - 9 = 0$  and using  $x = -3$ , is  $(-3)^2 - 9$  equal to  $0$ ? Yes! Therefore the solution is correct!

## c. Using the Square Root Property

Solve  $x^2 - 9 = 0$ .

Isolate the squared term

$$x^2 = 9$$

and use the *Square Root Property*

$$x = \pm\sqrt{9}$$

then  $x = 3$  or  $x = -3$ .

### Problem 10:

If  $3x^2 - 2x + 7 = 0$ , then  $(x - \frac{1}{3})^2$  is equal to what number?

This is tricky! The only option we have is to see if the quantity  $(x - \frac{1}{3})^2$  is hiding somewhere in  $3x^2 - 2x + 7 = 0$ .

First of all, let's multiply out  $(x - \frac{1}{3})^2$  to get

$$\begin{aligned}(x - \frac{1}{3})^2 &= (x - \frac{1}{3})(x - \frac{1}{3}) \\ &= x^2 - \frac{2}{3}x + \frac{1}{9}\end{aligned}$$

Certainly, we could write  $3x^2 - 2x + 7 = 0$  as  $3(x^2 - \frac{2}{3}x) + 7 = 0$ . That is, we factored a **3** out of the first two terms.

The terms enclosed in the parentheses now look like the first two terms of  $x^2 - \frac{2}{3}x + \frac{1}{9}$ .

So how could we insert a  $\frac{1}{9}$  without changing the value of the quadratic equation?

Watch this! If we were to insert a  $\frac{1}{9}$  into the parentheses then actually we changed the value of the left side by  $3(\frac{1}{9})$  because there is a 3 in front of the parentheses.

However, if we were to do the following we would not have changed the value of the left side at all:

$$3(x^2 - \frac{2}{3}x + \frac{1}{9}) + 7 - 3(\frac{1}{9}) = 0$$

Note that we both added and subtracted  $3(\frac{1}{9})$  on the left side. In effect, nothing happened except a change in appearance!

We can further write

$$\begin{aligned}3(x^2 - \frac{2}{3}x + \frac{1}{9}) + 7 - \frac{1}{3} &= 0 \\ 3(x - \frac{1}{3})^2 + \frac{20}{3} &= 0\end{aligned}$$

Again, we have not changed the value of the left side of the original equation

$$3x^2 - 2x + 7 = 0, \text{ just its appearance!}$$

Now, given  $3(x - \frac{1}{3})^2 + \frac{20}{3} = 0$ , we can solve for the quantity  $(x - \frac{1}{3})^2$  as follows:

$$3(x - \frac{1}{3})^2 = -\frac{20}{3}$$

$$\frac{1}{3}[3(x - \frac{1}{3})^2] = -\frac{20}{3}(\frac{1}{3})$$

and we find  $(x - \frac{1}{3})^2 = -\frac{20}{9}$

### Problem 11:

The monthly profit,  $P$ , in thousands of dollars, of a company can be estimated by the formula  $P = -3x^2 + 30x + 12$ , where  $x$  is the number of units produced and sold per month. Find the profit when 5 units are sold in one month.

$$P = -3(5)^2 + 30(5) + 12$$

$$P = 87$$

That is, when 5 units are sold in one month the profit is \$87.

### Problem 12:

A projectile is shot upward. It's distance  $s$  above the ground after  $t$  seconds is  $s = -16t^2 + 400t$ . Obviously, what goes up must come down! Calculate the time it takes for the projectile to return to the ground.

In physics, it is assumed that when  $s = 0$  the object is on the ground.

Thus,  $-16t^2 + 400t = 0$  will give us the time at which the projectile hits the ground.

Let's solve for  $t$  by factoring and then using the *Zero Product Principle* as follows:

$$-16t(t - 25) = 0$$

Then

$$-16t = 0$$

$$t = 0$$

This tells us that at  $t = 0$ , the object is on the ground!

$$t - 25 = 0$$

$$t = 25$$

From this calculation we find that the projectile is on the ground again after 25 seconds.