DETAILED SOLUTIONS AND CONCEPTS - POLYNOMIAL EQUATIONS

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Finding Solutions of Polynomial Equations

Factoring Method - Some, but not all polynomial equations can be solved by factoring. Later on in the course we will learn another method.

- Bring all terms of the polynomial equation to one side so that the other side is equal to 0. Combine like terms, if necessary.
- Factor relative to the integers.
- Apply the *Zero Product Principle*.

Problem 1:

Solve $2x^3 + 16x^2 + 30x = 0$. Find all solutions, real and imaginary!

First, we will factor out the greatest common factor.

$$2x(x^2 + 8x + 15) = 0$$

Then, we will factor the trinomial as follows:

$$2x(x+3)(x+5) = 0$$

Using the *Zero Product Principle*, we find

$2x = 0$ or $x + 3 = 0$ or $x + 5 = 0$

then $x = 0$ or $x = -3$ or $x = -5$.

We find that the polynomial equation has 3 real solutions!

Problem 2:

Solve $x^4 - 8x^2 - 9 = 0$. Find all solutions, real and imaginary!

Sometimes, you can encounter polynomial equations that are quadratic in form. That is, one exponent is exactly twice as large as the other exponent!

We can rewrite the first variable as follows:

$$(x^2)^2 - 8x^2 - 9 = 0$$

Then we factor.

$$(x^2 - 9)(x^2 + 1) = 0$$

Using the *Zero Product Principle* and using the *Square Root Property*, we get

$x^2 - 9 = 0$ or $x^2 + 1 = 0$

then $x^2 = 9$ or $x^2 = -1$

and $x = \pm 3$ or $x = \pm\sqrt{-1} = \pm i$.

We find that the polynomial equation has 2 real solutions and 2 imaginary solutions!

Problem 3:

Solve $x^3 - 8 = 0$. Find all solutions, real and imaginary!

Notice that we are dealing with a *Difference of Cubes*
$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$!

In our case, $a = 2$, therefore, we can factor as follows:

$$(x - 2)(x^2 + 2x + 4) = 0$$

Using the *Zero Product Principle* we will solve as follows.

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

The solution for the first factor is $x = 2$.

Using the *Quadratic Formula* for the second factor we get

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm i\sqrt{12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$$

We find that the polynomial equation has 1 real solution and 2 imaginary solutions!

Problem 4:

Solve $2x^3 + 3x^2 - 8x - 12 = 0$. Find all solutions, real and imaginary!

In this case, factoring is not readily apparent. Here, we will try to group two terms and then we will try to factor the common factor out of the first two terms and out of the last two! This is called *factoring by grouping*!

Let's group the first two terms and the last two terms:

$(2x^3 + 3x^2) + (-8x - 12) = 0$. Notice that you group by enclosing the terms in parenthesis. Observe how the last two terms were grouped!

We notice that the first two terms have a x^2 in common and the last two terms have **- 4**.

We will factor out the common factor of the first two terms and of the last two!

$$x^2(2x + 3) - 4(2x + 3) = 0$$

Notice that $(2x + 3)$ is a factor of the first term and the second term. Factor it out!

$$(2x + 3)(x^2 - 4) = 0$$

We notice that the second term is a *Difference of Squares* so that we can state the following:

$$(2x + 3)(x - 2)(x + 2) = 0$$

Using the *Zero Product Principle* we will solve as follows.

$2x + 3 = 0$ or $x - 2 = 0$ or $x + 2 = 0$

then $x = -\frac{3}{2}$ or $x = 2$ or $x = -2$.

We find that the polynomial equation has 3 real solutions!

Problem 5:

Solve $x^4 + 9x^2 = 0$. Find all solutions, real and imaginary!

First, we will factor out the greatest common factor.

$$x^2(x^2 + 9) = 0$$

Using the *Zero Product Principle*, we find

$x^2 = 0$ or $x^2 + 9 = 0$

Solving the first factor, we find that $x = 0$.

Solving the second factor using the *Square Root Property*, we find that

$x^2 = -9$, which results in $x = \pm\sqrt{-9} = \pm 3i$.

We find that the polynomial equation has 1 real solution and 2 imaginary solutions!

Problem 6:

Solve $x^4 - 16x^2 = 0$. Find all solutions, real and imaginary!

First, we will factor out the greatest common factor.

$x^2(x^2 - 16) = 0$

We notice that the second term is a *Difference of Squares* so that we can state the following:

$x^2(x - 4)(x + 4) = 0$

Using the *Zero Product Principle*, we find

$x^2 = 0$ or $x - 4 = 0$ or $x + 4 = 0$.

Solving the first factor, we get $x = 0$.

Solving the second and third factor, we get $x = 4$ and $x = -4$.

We find that the polynomial equation has 3 real solutions!

Problem 7:

Solve $x^3 + 27 = 0$. Find all solutions, real and imaginary!

Since the degree of the polynomial is three, we must find three *Zeros*, not necessarily distinct.

Notice that we are dealing with a *Sum of Cubes*
$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$!

In our case, ***a* = 3**, therefore, we can factor as follows:

$(x+3)(x^2-3x+9)=0$

Using the *Zero Product Principle* we set each factor equal to 0 and solve.

$x+3=0$ or $x^2-3x+9=0$

The solution for the first factor is $x=-3$.

Using the *Quadratic Formula* for the second factor we get

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2}$$

$$x=\frac{3\pm\sqrt{-27}}{2}=\frac{3\pm 3i\sqrt{3}}{2}.$$

We find that the polynomial equation has 1 real solution and 2 imaginary solutions!

Problem 8:

Solve $3x^3-x^2-6x+2=0$. Find all solutions, real and imaginary!

In this case, factoring is not readily apparent. Here, we will try to group two terms and then we will try to factor the common factor out of the first two terms and out of the last two! This is called *factoring by grouping*!

Let's group the first two terms and the last two terms:

$(3x^3-x^2)-(6x-2)=0$. Notice that you group by enclosing the terms in parenthesis. Observe how the last two terms were grouped!

We notice that the first two terms have a x^2 in common and the last two terms have -2.

We will factor out the common factor of the first two terms and of the last two!

$$x^2(3x-1)-2(3x-1)=0$$

Notice that $(3x - 1)$ is a factor of the first term and the second term. Factor it out!

$$(3x-1)(x^2-2)=0$$

Using the *Zero Product Principle* we will solve as follows:

$3x-1=0$ or $x^2-2=0$.

The solution for the first factor is $x = \frac{1}{3}$

and using the *Square Root Property* for the second factor we get

$$x^2 = 2$$
$$x = \pm\sqrt{2}.$$

We find that the polynomial equation has 3 real solutions!

Problem 9:

Solve $x^3 - x^2 - 6x = 0$. Find all solutions, real and imaginary!

First, we will factor out the greatest common factor.

$$x(x^2 - x - 6) = 0$$

Then, we will factor the trinomial as follows:

$$x(x+2)(x-3) = 0$$

Using the *Zero Product Principle*, we find

$x = 0$ or $x + 2 = 0$ or $x - 3 = 0$

then $x = 0$ or $x = -2$ or $x = 3$.

We find that the polynomial equation has 3 real solutions!