



DETAILED SOLUTIONS AND CONCEPTS - LINEAR EQUATIONS IN ONE VARIABLE

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Definition of a Linear Equation in one Variable

An equation containing one variable raised to the power of one (1) is called a *linear equation in one variable*.

For example, $2x + 3 = 11$.

Basic Principle of Equality

If an operation is performed on one side of the equal sign, the same operation must be performed on the other side. **This principle applies to ALL equations not just to linear equations.**

Addition Axiom

The same number can be added to both sides of an equation without changing the equality of the two sides. Since subtraction is the same as adding a negative number, this axiom also applies to subtraction. **This axiom applies to ALL equations not just to linear equations.**

Multiplication Axiom

Both sides of an equation can be multiplied by the same number without changing the equality of the two sides. **This axiom applies to ALL equations not just to linear equations.**

Strategy for Solving Linear Equations in one Variable:

- If necessary, use the *Distribute Property* to eliminate parentheses. Combine like terms.
- If necessary, bring every constant to one side of the equal sign using the *Addition Axiom*. Combine like terms.
- If necessary, bring every term containing a variable to the other side of the equal sign using the *Addition Axiom*. Combine like terms.
- If necessary, use the *Multiplication Axiom* to change the coefficient of the variable to a **positive 1**. That is, multiply both sides of the equal sign by the reciprocal of the coefficient.

Problem 1:

Solve the equation $y + 5 = 11$.

To solve the equation above, we will use the *Addition Axiom* to isolate the variable y by "moving" all terms associated with y by addition away from the variable.

The constant associated with y by addition is a **positive 5**. Therefore, we must add a **negative 5** to both sides of the equation.

$$y + 5 + (-5) = 11 + (-5)$$

Remember that we want to avoid double signs as much as possible. That's why we actually should just write

$$y + 5 - 5 = 11 - 5$$

NOTE:

ALWAYS use the Addition Axiom "inline", that is DO write the following:

$$y + 5 - 5 = 11 - 5$$

DO NOT write the value to be added to both sides below the equation. That is, DO NOT write the following:

$$y + 5 = 11$$
$$- 5 \quad - 5$$

Next, we will combine like terms, which **MUST** always be done in the final step.

We find that $y = 6$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $y + 5 = 11$ and using $y = 6$, is $6 + 5$ equal to 11 ? Yes! Therefore, the solution is correct!

Problem 2:

Solve the equation $y - 5 = 11$.

To solve the equation above, we will use the *Addition Axiom* to isolate the variable y by "moving" all terms associated with y by addition away from the variable.

We learned earlier to read a subtraction as "adding a negative number." Then the constant associated with y by addition is a **negative 5**.

Therefore, we must add a **positive 5** to both sides of the equation.

$$y - 5 + (+5) = 11 + (+5)$$

Again remember that we want to avoid double signs as much as possible. That's why we should actually write

$$y - 5 + 5 = 11 + 5$$

Lastly, we will combine like terms to get $y = 16$. This is the solution to the equation.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $y - 5 = 11$ and using $y = 16$, is $16 - 5$ equal to 11 ? Yes! Therefore, the solution is correct!

Problem 3:

Solve the equation $-9 + R = 3$.

In this case the constant associated with the variable by addition is a **negative 9**. Therefore, we must add a **positive 9** to both sides of the equation.

$$-9 + R + 9 = 3 + 9$$

Lastly, we will combine like terms to get $R = 12$.

Note that when expressing the solution, some beginning algebra students often switch to a lower case "r". In algebra, upper and lower case letters indicate distinctly different variables. If your equation contains an upper case variable, you MUST use it throughout the entire solution process. Likewise, if your equation contains a lower case variable, then you must use it throughout the entire solution process.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $-9 + R = 3$ and using $R = 12$, is $-9 + 12$ equal to 3 ? Yes! Therefore, the solution is correct!

Problem 4:

Solve the equation $3 = \frac{3}{2}x$.

We know that to solve equations, we must isolate the variable on one side of the equal sign. The variable in the example is not yet isolated because its coefficient must be a **positive 1**. We will use the *Multiplication Axiom* to change its coefficient.

The coefficient associated by multiplication with x is $\frac{3}{2}$. Therefore, we will multiply both sides of the equation by its **reciprocal** **, which is $\frac{2}{3}$.

** Interchanging the numerator and denominator of a fraction results in a fraction that is called the **reciprocal** of the original fraction. When a number is multiplied by its reciprocal, the product equals **1**.

$$\left(\frac{2}{3}\right)(3) = \left(\frac{2}{3}\right)\left(\frac{3}{2}x\right)$$

Carrying out the multiplications, we get

$$2 = x \text{ which should be presented as } x = 2$$

Note: When solving equation, it is customary to show the solution with the variable on the left side of the equal sign. However, it is NOT WRONG to show the variable on the right.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $3 = \frac{3}{2}x$ and using $x = 2$, is $\frac{3}{2}(2)$ equal to **3**? Yes!
Therefore, the solution is correct!

Problem 5:

Solve the equation $-18 = 2x$.

The coefficient of x is **2**. Therefore, we will multiply both sides of the equation by its reciprocal, which is $\frac{1}{2}$.

$$\left(\frac{1}{2}\right)(-18) = \left(\frac{1}{2}\right)(2x)$$

Carrying out the multiplications, we get

$$-9 = x \text{ which should be presented as } x = -9$$

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $-18 = 2x$ and using $x = -9$, is $2(-9)$ equal to -18 ? Yes! Therefore, the solution is correct!

Problem 6:

Solve the equation $\frac{x}{3} = -9$.

In this case, the coefficient of x is somewhat "disguised." What we need to know is that we can write

$$\frac{x}{3} \text{ as } \frac{1}{3}x.$$

Now we can see that the coefficient is $\frac{1}{3}$ so that we can write

$$\frac{1}{3}x = -9$$

Then $\left(\frac{3}{1}\right)\left(\frac{1}{3}x\right) = \left(\frac{3}{1}\right)(-9)$ and carrying out the multiplications, we get $x = -27$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $\frac{x}{3} = -9$ and using $x = -27$, is $\frac{-27}{3}$ equal to -9 ? Yes! Therefore, the solution is correct!

Problem 7:

Solve the equation $\frac{3x}{4} = 18$.

In this case, the coefficient of x is again somewhat "disguised." What we need to know is that we can write

$$\frac{3x}{4} \text{ as } \frac{3}{4}x.$$

That is, we can rewrite the equation as follows:

$$\frac{3}{4}x = 18$$

Then $\left(\frac{4}{3}\right)\left(\frac{3}{4}x\right) = \left(\frac{4}{3}\right)(18)$ and carrying out the multiplications, we get $x = 24$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $\frac{3x}{4} = 18$ and using $x = 24$, is $\frac{3(24)}{4}$ equal to 18 ? Yes! Therefore, the solution is correct!

Problem 8:

Solve the equation $\frac{t+9}{4} = -1$.

Here it is best to "get rid" of the fraction entirely by simply multiplying both sides of the equation by 4 to "eliminate" the denominator.

$$\left(\frac{4}{1}\right)\left(\frac{t+9}{4}\right) = \left(\frac{4}{1}\right)(-1)$$

We can now cross-cancel 4 on the left side of the equal sign and multiply on the right to get

$$t + 9 = -4$$

and using the *Addition Axiom* we can write $t = -13$

Please note that it is perfectly okay to use the Addition and Multiplication Axioms mentally! You should learn to do this as much as possible, but staying within your own comfort zone! It can cut out a lot of solution steps!

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $\frac{t+9}{4} = -1$ and using $x = -13$, is $\frac{-13+9}{4}$ equal to -1 ? Yes! Therefore, the solution is correct!

Problem 9:

Solve the equation $-x = 10$.

This equation is not yet solved. The coefficient of the variable must be a **positive 1** for the equation to be solved.

Therefore, we must multiply both sides of the equal sign by **-1**, which is the reciprocal of **-1**.

$$-x(-1) = 10(-1)$$

Carrying out the multiplications, we get $x = -10$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $-x = 10$ and using $x = -10$, is $-(-10)$ equal to 10 ? Yes! Therefore, the solution is correct!

Problem 10:

Solve the equation $-18 = -2.4x$.

The coefficient of x is **-2.4**. Therefore, we will multiply both sides of the equation by its reciprocal, which is $-\frac{1}{2.4}$.

$$\left(-\frac{1}{2.4}\right)(-18) = \left(-\frac{1}{2.4}\right)(-2.4x)$$

Carrying out the multiplications, we get

$$7.5 = x \text{ which should be presented as } x = 7.5$$

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $-18 = -2.4x$ and using $x = 7.5$, is $-2.4(7.5)$ equal to -18 ? Yes! Therefore, the solution is correct!

Problem 11:

Solve $8x + 12 = 28$.

Here we have constants on either side of the equal sign. Since our ultimate goal is to isolate the variable, it is best to have all constants on the right (left) of the equal sign using the *Addition Axiom*.

$$8x = 16$$

Now we'll use the *Multiplication Axiom* and multiply both sides of the equation by the reciprocal of the coefficient to find that $x = 2$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $8x + 12 = 28$ and using $x = 2$, is $8(2) + 12$ equal to 28 ? Yes! Therefore, the solution is correct!

Problem 12:

Solve $2a - 5 = 10 - 3a$.

Again we want all constants on the right side of the equal sign, but we also want all variables on the left side or vice versa. Using the *Addition Axiom*, we get

$$2a = 15 - 3a$$

and

$$5a = 15$$

NOTE: You could have carried out the "movement" of the terms in one step!

Finally, using the *Multiplication Axiom* and multiplying both sides of the equation by the reciprocal of the coefficient, we find that $a = 3$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $2a - 5 = 10 - 3a$ and using $a = 3$, is $2(3) - 5 = 10 - 3(3)$? Yes! Therefore, the solution is correct!

Problem 13:

Solve $2x - 3 = 9 - 10x$.

Using the *Addition Axiom*, we get

$$2x = 12 - 10x$$

and

$$12x = 12$$

NOTE: Again, you could have carried out the "movement" of the terms in one step.

Finally, using the *Multiplication Axiom* and multiplying both sides of the equation by the reciprocal of the coefficient, we find that $x = 1$.

Check:

Please note that if we check this solution in the original equation, we find the following:

Given $2x - 3 = 9 - 10x$ and using $x = 1$, is $2(1) - 3 = 9 - 10(1)$? Yes!
Therefore, the solution is correct!

Problem 14:

Solve $9 - 4x = 8x$.

Using the *Addition Axiom*, we get

$$9 = 12x$$

Finally, using the *Multiplication Axiom* and multiplying both sides of the equation by the

reciprocal of the coefficient, we find that $\frac{9}{12} = x$.

Note that a solution is always presented in reduced form, therefore, we must say

$$x = \frac{3}{4}$$

NOTE: Unless the equation already contains decimal numbers, it is customary to express the solution as a fraction. That is, we would NOT say $x = 0.75$.

Problem 15:

$$\text{Solve } 7 - 3x = 11.$$

Using the *Addition Axiom*, we get

$$- 3x = 4$$

Finally, using the *Multiplication Axiom* and multiplying both sides of the equation by the reciprocal of the coefficient, we find that

$$x = -\frac{4}{3}$$

Again, unless the equation already contains decimal numbers, it is customary to express the solution as a fraction, even if it is improper. That is, we would NOT have stated $x = -1.33$.

Problem 16:

$$\text{Solve } 3x + 2(4 - 9x) - 3(x - 3) + x = 0.$$

First use the *Distributive Property of Multiplication* to eliminate parentheses.

This means that we can write the given equation in simplified form as follows:

$$3x + 8 - 18x - 3x + 9 + x = 0$$

Please note that ALL calculations were carried out at the same time writing a complete equation. Sometimes students will add "helper" calculations to the solution path that are not complete equations. This practice is NOT customary and unacceptable!

Next, we'll combine like terms

$$-17x + 17 = 0$$

then we separate the variable and the constant

$$-17x = -17$$

and finally, we isolate the variable by multiplying both sides of the equality by the reciprocal of the coefficient of the variable to find that $x = 1$.

Problem 17:

$$\text{Solve } 7 - (x - 8) = 4x.$$

First we use the *Distributive Property of Multiplication* to eliminate parentheses.

NOTE: Here you must know that we have a product $-(x - 8)$, where one factor is the constant -1 . So we actually multiply each term in the second factor by -1 as follows:

$$\begin{aligned} -(x - 8) &= -1(x) - 1(-8) \\ &= -x + 8 \end{aligned}$$

This means that we can write our equation in simplified form as follows:

$$7 - x + 8 = 4x$$

Next, we'll combine like terms on the left

$$15 - x = 4x$$

then we separate the variable and the constant to get

$$15 = 5x$$

and finally, we isolate the variable using the *Multiplication Axiom* to get $3 = x$ which is presented as $x = 3$.

Problem 18:

$$\text{Solve } \frac{3}{4} - x = \frac{7}{8}.$$

We will solve this equation using two different solution paths.

Solution Method 1:

Again we want all constant terms on the right of the equal sign. Using the *Addition Axiom*, we get

$$-x = \frac{7}{8} - \frac{3}{4}$$

NOTE: Given fraction, it is a good idea to write the *Addition Axiom* out (at least on the right side) and not do mental calculations!

Now we have to combine like terms on the right using the **LCD 8**. Then

$$-x = \frac{7}{8} - \frac{6}{8} \quad \text{and} \quad -x = \frac{1}{8}$$

Please note that the variable is negative!!! Just because we "moved a constant across the equal sign" doesn't mean that we can "lose" the negative sign of the variable.

We actually now have to use the *Multiplication Axiom* to finish solving the equation. Since the coefficient of the variable must be **+1** for the equation to be solved, we must multiply both sides of the equal sign by **-1**, which is the reciprocal of **-1**

and we find that $x = -\frac{1}{8}$.

Solution Method 2:

Let's solve $\frac{3}{4} - x = \frac{7}{8}$ again, but this time we'll use a different solution path.

Some individuals feel that it is best to first "get rid of pesky fractions" before solving the equation. You do this by multiplying both sides of the equation by the LCD of ALL the denominators in the equation.

We can see that the **LCD** of ALL of the denominators is **8**. Therefore,

$$8\left(\frac{3}{4} - x\right) = 8\left(\frac{7}{8}\right)$$

and using the *Distributive Property* on the left side of the equal sign, we get

$$8\left(\frac{3}{4}\right) - 8x = 8\left(\frac{7}{8}\right)$$

After we multiply, we end up with

$$6 - 8x = 7$$

Then, using the *Addition Axiom*, we find

$$-8x = 1$$

and using the *Multiplication Axiom* and multiplying both sides of the equation by the reciprocal of the coefficient of x , we get

$$x = -\frac{1}{8}$$

Problem 19:

Solve $5x - 15 = 5(x - 3)$.

First we use the *Distributive Property of Multiplication* to eliminate parentheses.

$$5x - 15 = 5x - 15$$

Then we separate the variable and the constant

$$-15 = -15$$

We see that the variable has "disappeared." However, since the last statement is true, any number is a solution for this equation. Here we say that this equation has **infinitely many** solutions!

Problem 20:

Solve $2x + 3(x + 1) = 5x + 4$.

First we use the *Distributive Property of Multiplication* to eliminate parentheses.

$$2x + 3x + 3 = 5x + 4$$

then $5x + 3 = 5x + 4$

Finally, we separate the variable and the constant to find

$$0 = 1$$

Again, the variable has "disappeared", but this time the last statement is false. When this happens, we say that this equation has **NO** solutions!

Problem 21:

Solve $x + \frac{1}{7}x = 16$.

Here it is best to "get rid" of the fraction entirely by simply multiplying both sides of the equation by 7 to "eliminate" the denominator.

$$7\left(x + \frac{1}{7}x\right) = 7(16)$$

and using the *Distributive Property of Multiplication*, we get

$$7x + 7\left(\frac{1}{7}\right)x = 112$$

which simplifies to $7x + x = 112$ or $8x = 112$

Finally, using the *Multiplication Axiom* and multiplying both sides of the equation by the reciprocal of the coefficient of x , we find that $x = 14$.

Please note that it is perfectly okay to use the Addition and Multiplication Axioms mentally! You should learn to do this as much as possible, but staying within your own comfort zone! It can cut out a lot of solution steps!