DETAILED SOLUTIONS AND CONCEPTS - THE LAWS OF EXPONENTS

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

The Laws of Exponents

Please be aware that the letters *a*, *b*, *m* , and *n* are replacements for any real number. However, when the letters are identical, we must use the SAME number replacement!

$a^m a^n = a^{m+n}$ When an exponential expression is multiplied by another exponential expression **having the same base**, the powers are added.	$a^0 = 1,\ a \neq 0$ Any number, except for 0, raised to the zero power results in a value of 1.
$\frac{a^m}{a^n} = a^{m-n}$ When an exponential expression is divided by another exponential expression **having the same base**, the power in the denominator is subtracted from the power in the numerator.	$(ab)^m = a^m b^m$ When a product is raised to a power, each factor is raised to the power. $\left(a^m\right)^n = a^{m(n)}$ When an exponential expression is raised to a power, the powers are multiplied.
$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$ When a number is raised to a negative power, the exponential expression can be placed in the denominator of a fraction with numerator 1, but the negative sign in the exponent changes to a positive sign.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ When a fraction is raised to a power, this power can be distributed to the numerator and to the denominator.

Problem 1:

Multiply $x^2 \cdot x^3$

$$x^2 \cdot x^3 = x^{2+3} = x^5$$

Problem 2:

Multiply $x \cdot x^6$

$$x \cdot x^6 = x^{1+6} = x^7$$

Problem 3:

Multiply $10^2 \cdot 10^5$

$$10^2 \cdot 10^5 = 10^{2+5} = 10^7 = 10{,}000{,}000$$

NOTE: $10^2 \cdot 10^5 \neq 100^{2+5}$

Problem 4:

Multiply $10^2 \cdot 6^5$

$$10^2 \cdot 6^5 = 10^2 \cdot 6^5 = 100 \cdot 7{,}776 = 777{,}600$$.

NOTE: $10^2 \cdot 6^5 \neq 60^{2+5}$

Please note that the law tells us that we have to have identical numbers in the base before we can add the exponents.

Problem 5:

Multiply $2xy^2(-3xy^2)$.

When you are multiplying two or more terms containing variables, the operation becomes easier if you group together the numbers and the exponential expressions with like base and like powers as follows:

$$\begin{aligned} 2xy^2(-3xy^2) &= 2(-3) \cdot x \cdot x \cdot y^2 \cdot y^2 \\ &= -6x^2y^4 \end{aligned}$$

NOTE: You do not have to write down the "grouping" step. Instead you can write the answer right away.

Problem 6:

Multiply $-2(3a)(-5bc^2)(-2ac)$.

This multiplication becomes easier to group as follows:

$$
\begin{aligned}
-2(3a)(-5bc^2)(-2ac) &= -2(3)(-5)(-2)\cdot a\cdot a\cdot b\cdot c^2\cdot c \\
&= -60a^2bc^3
\end{aligned}
$$

Problem 7:

Multiply $2(10a)$.

$$
\begin{aligned}
2(10a) &= 2(10)\cdot a \\
&= 20a
\end{aligned}
$$

Problem 8:

Divide $\frac{x^5}{x^2}$.

$$\frac{x^5}{x^2} = x^{5-2} = x^3.$$

Problem 9:

Divide $\frac{x^6}{x}$

$$\frac{x^6}{x} = x^{6-1} = x^5.$$

Problem 10:

Divide $\frac{10^5}{10^2}$.

$$\frac{10^5}{10^2} = 10^{5-2} = 10^3 = 1{,}000.$$

NOTE: $\frac{10^5}{10^2} \neq 1^{5-2}$

Problem 11:

Divide $\frac{6^4}{3^2}$.

$$\frac{6^4}{3^2} = \frac{6^4}{3^2} = \frac{1,296}{9} = 144$$.

NOTE: $\frac{6^4}{3^2} \neq 2^{4-2}$

Problem 12:

Divide $\frac{x^3}{x^3}$.

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$.

This immediately allows us to illustrate the law $a^0 = 1,\ a \neq 0$, which states that any number raised to the zero power results in a value of 1.

Therefore, $x^0 = 1$. Please note that $x^0 \neq 0$.

Problem 13:

Find the value of 5^0.

$5^0 = 1$

Problem 14:

Find the value of $1,000,000^0$.

$1,000,000^0 = 1$

Problem 15:

Find the values of $(-2)^0$ and -2^0.

$(-2)^0 = 1$

Please note that $-2^0 = -1(2^0) = -1(1) = -1$.

By the *Order of Operation*, exponential expressions are simplified **BEFORE** we multiply (in this case by -1)!

Problem 16:

Divide $\frac{x^2}{x^5}$.

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3}$$
.

This immediately allows us to illustrate the law $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$, where the negative exponent indicates that the exponential expression is actually in the denominator of a fraction with numerator 1. Then, when written as a fraction, the negative sign in the exponent changes to a positive sign.

Therefore, $x^{-3} = \frac{1}{x^3}$. Please note that $x^{-3} \neq -x^3$.

Problem 17:

Rewrite in terms of positive exponents: y^{-4}

$$y^{-4} = \frac{1}{y^4}$$

Problem 18:

Rewrite in terms of positive exponents: 3^{-3}

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

Problem 19:

Rewrite in terms of positive exponents: $(-4)^{-3}$ and -4^{-3}.

$$(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} \text{ or } -\frac{1}{64}$$

Now observe, $-4^{-3} = -\frac{1}{4^3} = -\frac{1}{64}$.

Here we actually have $\boldsymbol{-1(4)^{-3}}$ and we have to go by the *Order of Operation*.

Problem 20:

Rewrite in terms of positive exponents: $(-4)^{-4}$ and -4^{-4}.

$$(-4)^{-4} = \frac{1}{(-4)^4} = \frac{1}{256}$$

Now observe, $-4^{-4} = -\frac{1}{4^4} = -\frac{1}{256}$

Here we actually have $\boldsymbol{-1(4)^{-4}}$ and we have to go by the *Order of Operation*.

Problem 21:

Divide $\dfrac{-12x^3y^5}{3xy^2}$.

Here we must group together the numbers and the exponential expressions with like base as follows:

$$\frac{-12x^3y^5}{3xy^2} = \frac{-12}{3} \cdot \frac{x^3}{x} \cdot \frac{y^5}{y^2}$$
$$= -4x^2y^3$$

NOTE: You do not have to write down the "grouping" step. Instead you can write the answer right away.

Problem 22:

Divide $\frac{8ab^3}{2}$.

$$\frac{8ab^3}{2} = \frac{8}{2} \cdot \frac{a}{1} \cdot \frac{b^3}{1}$$
$$= 4ab^3$$

Problem 23:

Divide $\frac{-18x^3y^{-9}}{3x^2y^2}$. Write your answer with positive exponents only!

$$\frac{-18x^3y^{-9}}{3x^2y^2} = \frac{-18}{3} \cdot \frac{x^3}{x^2} \cdot \frac{y^{-9}}{y^2}$$
$$= -6xy^{-9-2}$$
$$= -6xy^{-11}$$
$$= \frac{-6x}{y^{11}}$$

Problem 24:

Divide $\frac{4ab^5}{3ab^{-4}}$.

$$\frac{4ab^5}{3ab^{-4}} = \frac{4}{3} \cdot \frac{a}{a} \cdot \frac{b^5}{b^{-4}}$$
$$= \frac{4}{3} \cdot 1 \cdot b^{5-(-4)}$$
$$= \frac{4}{3} b^{5+4}$$
$$= \frac{4}{3} b^9 \text{ or } \frac{4b^9}{3}$$

Problem 25:

Simplify $\left(\frac{2}{x}\right)^3$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we will be asked to "simplify" instead of finding the value of the number raised to the third power.

$$\left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3} = \frac{8}{x^3}$$

Problem 26:

Simplify $\left(\frac{3}{4}\right)^2$.

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Problem 27:

Simplify $\left(\frac{5}{2}\right)^3$.

$$\left(\frac{5}{2}\right)^3 = \frac{5^3}{2^3} = \frac{125}{8}$$

Problem 28:

Simplify $\left(\frac{1}{3}\right)^4$.

$$\left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \frac{1}{81}$$

Note that the number 1 raised to any power will always have a value of 1.

Problem 29:

Simplify $\left(\frac{-3}{5}\right)^2$.

$$\left(\frac{-3}{5}\right)^2 = \frac{(-3)^2}{5^2} = \frac{9}{25}$$

Problem 30:

Simplify $(x^2)^3$.

$$(x^2)^3 = x^{2(3)} = x^6$$

Note that $x^2 \cdot x^3 = x^{2+3} = x^5$.

Problem 31:

Simplify $(5x)^2$.

$$(5x)^2 = 5^2 x^2 = 25x^2$$

Please note that $(5+x)^2 \neq 5^2 + x^2$ and $(5-x)^2 \neq 5^2 - x^2$. Later on we will learn how to deal with sums and differences raised to a power!

Problem 32:

Simplify $(-3a^3b^4c)^2$.

$$\begin{aligned}(-3a^3b^4c)^2 &= (-3)^2(a^3)^2(b^4)^2(c)^2 \\ &= 9a^6b^8c^2\end{aligned}$$

Please note that this law extends to any product containing infinitely many factors.

Problem 33:

Simplify $(-5h^{-1}k^{-2})^{-3}$.

then $(-5)^{-3}(h^{-1})^{-3}(k^{-2})^{-3}$

and $-\frac{1}{5^3}h^3k^6 = -\frac{1}{125}h^3k^6$